The solution of integrable quantum models by the complete characterization of their spectrum and dynamics is one of the fundamental issues in mathematical physics as it allows achieving exact (non-perturbative) results in several areas of physics where these models play a central role. Despite significant progress obtained in the last forty years for some lattice models (like the Heisenberg spin chains), thanks to the existence of powerful algebraic structures related to the Yang-Baxter algebra, the full solution of more general integrable quantum field theories (IQFTs) is still a fundamental open problem in mathematical physics.

We present a microscopic approach in the framework of Sklyanin’s quantum separation of variables (SOV) for the exact solution of 1+1 dimensional quantum field theories by integrable lattice regularizations. Sklyanin’s SOV is the natural quantum analogue of the classical method of separation of variables and it allows a more symmetric description of classical and quantum integrability w.r.t. traditional Bethe ansatz methods. Moreover, it has the advantage to be applicable to a more general class of models for which its implementation gives a characterization of the spectrum complete by construction. Our aim is to show how from the implementation of the SOV both the spectrum (eigenvalues and eigenvectors) and the dynamics (correlation functions) of IQFTs are derived.

This approach is presented for a paradigmatic example of relativistic IQFT, the sine-Gordon model, focusing the attention on the complete SOV description of its spectrum; a short overview of the first fundamental results toward the dynamics is also provided. These results allow reformulating the spectrum and dynamics characterization in terms of a well defined mathematical problem, i.e. the classification of the solutions to a functional equation (Baxter’s equation) in a fixed class of functions. Finally, the potential generality of the method emerging from its implementation for a series of key integrable quantum models is also discussed.

Keywords: Integrable quantum models, quantum inverse scattering method, Sklyanin’s separation of variables.